

MCS 452 MT2 Questions

- 1) a) Give the definition of an inner product space,
b) Show that the space ℓ^3 is not an inner product space and hence not a Hilbert space.

- 2) Let X be an inner product space. Suppose x and y are orthogonal vectors in X . Show that

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2$$

- 3) Show that an orthogonal set of non-zero vectors in X is linearly independent.

- 4) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ $T(x, y, z) = (\frac{1}{5} \sin x, \frac{1}{4} \sin y, \frac{\pi}{2} + \frac{1}{3} \cos z)$

a) Show that T is a contraction in usual metric.

b) Show that $u = (0, 0, \frac{\pi}{2})$ is a solution of the equation $Tu = u$. c) Is there another solution of the equation $Tu = u$? Describe your answer.

- 5) Show that if Y is a closed subspace of a Hilbert space H , then $Y = Y^{\perp\perp}$.