

MCS 452 MT1 Questions

1) Consider the linear operator $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ defined by

$$T(x, y, z, w) = (2x + 3y, 4y - 7z + 2w)$$

Find the kernel (null space) of the operator. Show that the dimension of the range of T is 2.

2) TRUE or FALSE? If TRUE, give proof; if FALSE, give a counterexample.

(a) Every convergent sequence in a metric space is a Cauchy sequence.

(b) If $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent norms on X , the Cauchy sequences in $(X, \|\cdot\|_1)$ and $(X, \|\cdot\|_2)$ are the same.

(c) The space of rational numbers, \mathbb{Q} , is complete.

(d) The space of all Riemann integrable functions on $[a, b]$ is a normed space.

3) If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in a metric space (X, d) , then the sequence $\{a_n\}$ converges, where $a_n = d(x_n, y_n)$.

4)(a) Show that $f(x) = \frac{x}{x+1}$ is an increasing function on whole of its domain.

(b) If $d(x, y)$ is a metric in X , show that $\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ is also a metric in X .