# MCS 452-HW1

**Q1**. Consider the space $C[0,5]$ of continuous real-valued functions on the closed interval $[0,5]$. Let $f(x) = x^2 - 4x, g(x) = 3x - 6$. What is $d(f,g)$ if $d$ is
(a) $d_1$ (**the supremum metric**), and (b) $d_2$ (**the area metric**)?

**Q2**. Recall that $(C[0,1]; d_1)$ is the metric space of continuous functions $f : [0,1] \to \mathbb{R}$ with the metric $d_1(f,g) = \sup\{|f(x) - g(x)| : x \in [0,1]\}$.
Let $f \in C[0,1]$ be the constant function with value 0, i.e. $f(x) = 0$ for all $x$. Describe the closed ball $B[f;1]$.

**Q3**. Prove that the function $d$ defined by $d(x,y) = \sqrt{|x-y|}$ (where $x, y \in \mathbb{R}$) is a metric on the set $\mathbb{R}$.
Prove that the function $d$ defined by $d(x,y) = (x-y)^2$ (where $x, y \in \mathbb{R}$) is not a metric on the set $\mathbb{R}$.

**Q4**. Show that the sequence $(x_n, y_n) = (\sin(\frac{1}{1+n^2}), \sqrt{\frac{1}{n+n^4}})$ converges to $(0,0) \in \mathbb{R}^2$, with respect to each of the three metrics $d_1, d_2$ and $d_1$. (Hint: Use the fact that $|\sin x| \leq |x|$ for all real numbers $x$).

**Q5**. Let $(x_n)$ and $(y_n)$ be two sequences in a metric space $(X,d)$.
(a) Show that if $x_n \to a$ and $y_n \to a$ then $d(x_n, y_n) \to 0$.
(b) Show that if $x_n \to a$ and $d(x_n, y_n) \to 0$ then $y_n \to a$.
(c) Show that if $x_n \to a$ and $y_n \to b$ then $d(x_n, y_n) \to d(a,b)$.
(Hint: Use the identity $|d(x,y) - d(a,b)| \leq d(x,a) + d(y,b)$.)

**Q6**. Prove that the set of all irrational numbers are not closed in $\mathbb{R}$

**Q7**. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^2 + y^2$. What is $f^{-1}[1,2]$? Sketch a picture of this set.